A Comparison Between OFDM With TZCP Guard Time vs. Cyclic Prefix & Trailing Zeros

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Abstract- **In this paper, the transmission of an orthogonal frequency division multiplexed (OFDM) signal over channel with time varying frequency has been considered, selective fading and additive white Gaussian noise (AWGN). Due to channel dispersion, intersymbol and intercarrier Interference will occur at the receiver. To reduce this interference, a guard time is added as a cyclic prefix (CP), or trailing zeros (TZ). This paper presented a novel combination of two previous guard time, named TZCP. Also the system performance for CP, TZ and TZCP guard time has been simulated and implemented over slow fading and HF channel and has shown better performance obtained in both channels for TZCP gaurdtime.**

Keywords-component; OFDM; TZCP; TZ; TZ; HF channel; fading channel

I. Introduction

Wireless communications is beginning to exercise a great influence in our daily lives. The enormous increase of wireless and portable telephones is leading to a number of initiatives for new systems and services. In the future, we will be witness a widespread deployment in diversified wireless services [1].
Because of the increasing demand of wireless communications, there is a need of a modulation technique that can transmit reliably high data rates at a high bandwidth efficiency. An excellent candidate is orthogonal frequency division multiplexing (OFDM) [3, 4]. This technique has been developed in the 60's but only recently the implementation became technical feasible and cost competitive by means of DSP technology. Under the name discrete multitone (DMT), it is used as basis for a worldwide XDSL standard and more recently for small area mobile wireless broadband systems (ETSI BRAN HiperLAN/2. IEEE 802.11a and MMAC) OFDM is an effective method to combat interference in signaling over multipath radio channels [8].
OFDM partitions a given bandwidth in a set of orthogonal subchannels. These subchannels can be modulated and demodulated using discrete Fourier transforms (DFT). In practice, the DFT are implemented by efficient fast Fourier transform (FFT) techniques. Interference, caused by the time dispersive channel is reduced by adding a guard time that consists of a cyclic prefix. Although inserting this guard time reduces interference but at the same time, it decreases transmission efficiency.
In this paper, the influence of the guard time on the system performance is investigated, and then mathematical models for CP and TZ guard time is described. By regarding benefits of two said methods, a novel guard time presented named it TZCP. Also by performing simulations performed by matlab software, BER result between different methods was compared.

II. An OFDM transceiver

Orthogonal frequency division multiplexing (OFDM) is a modulation technique that results in a high bandwidth efficiency and a relatively low complexity. OFDM partitions the given bandwidth in equidistant orthogonal subchannels, which can be modulated and demodulated easily using discrete Fourier transforms (DFT).
In Fig. 1, an OFDM transceiver is depicted.

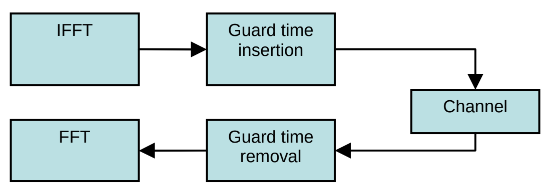

Figure 1- An OFDM Transceiver

A set of N subsymbols $a_{i,n}$, where the index n[0,N-1] refers to the n^{th} subchannel and the index i to the i^{th} transmitted symbol, is modulated using an inverse DFT. To reduce the interference, caused by the time-dispersive channels, a guard time interval v is added as a cyclic prefix. The transmitted samples $g_i(m)$, corresponding to the i^{th} symbol interval, are given by:

$$s_i(m) = \sqrt{\frac{Es}{N+v}} \sum_{n=0}^{N-1} S_{i,n}(f) \exp(j 2\pi \frac{nm}{N})$$

m=-v…N-1　(1)

The subsymbols $a_{i,n}$ are statistically independent and have a unit average energy:

$$E[S^*_{i,n}\, S_{j,m}] = \delta_{i,j}\delta_{n,m} \quad (2)$$

At the receiver, the cyclic prefix is removed before demodulation by a DFT. A frequency domain equalizer, not shown in figure 1, follows the DFT. In practice, the DFT's are implemented by Fast Fourier Transforms (FFT).

III. Channel description

In wireless communications, different objects block signal propagation and the signal power is carried by a large amount of paths with different strengths and delays. We used a statistical analysis for simplicity. The resulting received signal, due to the summation of all reflected signals, can be approximated using the central limit theorem by a complex Gaussian random variable. The amplitude of this complex Gaussian variable has a Ricean distribution or, if the mean value is zero (no direct paths), a Rayleigh distribution. The characteristics of the transmission media are constantly changing and involve randomly time-varying impulse responses *(k ; l)*

$$x_{out}(k) = \sum_{l=-\infty}^{+\infty} x_{in}(k-l)h(k;l) \quad (3)$$

Where $x_{in}(k)$ are the input samples and $x_{out}(k)$ are the output samples. The impulse response is described by its autocorrelation function:

$$R_{hh}(k_1,k_2;l_1,l_2) = E[h(k_1,l_1)h^*(k_2,l_2)] = \delta(k_1-k_2)R(k_1;l_1-l_2) \quad (4)$$

The wide-sense stationary uncorrelated scattering (WSSUS) model [1] makes physical assumptions that are valid for most radio transmission channels:

- The signal variations on paths arriving at different delays are uncorrelated
- The correlation properties of the channel are stationary.

With this WSSUS model, the autocorrelation function becomes:

$$R_{hh}(k_1,k_2;l_1,l_2) = R(k_1;l_1-l_2)\delta(k_1-k_2) \quad (5)$$

In what follows, we assume that the transmitted samples are only disturbed by WSSUS multipath time-varying frequency-selective Rayleigh fading and additive white Gaussian noise (AWGN).

IV. Performance

At the input of the receiver, we obtain the samples *z(l)*, given by:

$$z(\ell) = \sqrt{\frac{E_s}{N+v}} \sum_{i=-\infty}^{+\infty}\sum_{n=0}^{N-1}\sum_{m=-v}^{N-1}\left(a_{i,n} e^{j2\pi\frac{nm}{N}} h(\ell-m-i(N+v);\ell) \right) + W(\ell) \quad (6)$$

Where *W(l)* is white Gaussian noise with independent real and imaginary parts, each having a variance of $N_0/2$. Assuming that the impulse response h(k;l) has an average energy equal to l, we get:

$$\sum_{k=-\infty}^{+\infty} R(K;0) = 1 \quad (7)$$

At the output of the FFT we find:

$$W_K^{(j)} = \sqrt{\frac{E_s}{N(N+v)}} \sum_{i=-\infty}^{+\infty}\sum_{n=0}^{N-1} a_{i,n}\gamma_{i,k,n} + w_k \quad (8)$$

Where

$$\gamma_{i,k,n} = \sum_{m=-v}^{N-1}\sum_{\ell=0}^{N-1}\left(e^{-j2\pi\frac{k\ell-nm}{N}} \quad h((\ell-m)-i(N+v);\ell) \right) \quad (9)$$

The average total power *p(k)* at the output *k* of the FFT is given by:

$$P(K) = E\left[\left|W_k^{(j)}\right|^2\right] = \frac{E}{N(N+v)} E\left[\left|\gamma_{0,k,k}\right|^2\right] + \frac{E_s}{N(N+v)}\sum_{\substack{n=0\\n\neq k}}^{N-1} E\left[\left|\gamma_{0,k,n}\right|^2\right] + \frac{E_s}{N(N+v)}\sum_{\substack{i=-\infty\\i\neq 0}}^{+\infty}\sum_{n=0}^{N-1} E\left[\left|\gamma_{0,k,n}\right|^2\right] + N_0 \quad (10)$$

The average total power consists of 4 contributions. The first component is the useful power. Besides the useful power, we find intercarrier interference (ICI) and intersymbol interference (ISI). The last contribution is white Gaussian noise.

Assuming that the impulse response *h(k; l)* has a limited duration:

$$h(k;l) = 0 \quad \text{k<0 and } k \geq N+v \quad (11)$$

The summation over i in the ISI contribution reduces to i=-1, which means that only the previous symbol interferes during the present symbol interval.

We define $P_U(k)$ and $P_I(k)$ as the normalized useful power and the total interference power, respectively:

$$P_U = (K)\frac{1}{N^2}E\left[\left|\gamma_{o,k,k}\right|^2\right] \quad P_I = (K)\frac{1}{N^2}\sum_{\substack{n=0\\n\neq 0}}^{N-1} E\left[\left|\gamma_{o,k,n}\right|^2\right] + \frac{1}{N^2}\sum_{n=0}^{N-1}E\left[\left|\gamma_{-1,k,n}\right|^2\right] \quad (12)$$

The total power *P(k)* becomes:

$$P(K) = E_S \frac{N}{N+v} P_U(K) E_S \frac{N}{N+v} P_I(K) + N_0 \quad (13)$$

When all carriers are modulated and using assumption (7), we can find that $P_U(k)$ are independent of the index k, and that $P_U(k)+P_I(k)=1$. In the following, we drop the index k. we obtain the signal-to-noise ratio (SNR) at the output of the FFT:

$$SNR = \frac{E_s \frac{N}{N+v} p_u}{E_s \frac{N}{N+v}(p_{ICI} + p_{ISI}) + N_0} \quad (14)$$

We define the degradation of the SNR as

$$Deg = -10 \log \left(\frac{\frac{N}{N+v} p_u}{1 + \frac{E_s}{N_0} \frac{N}{N+v}(1 - p_u)} \right) \quad (15)$$

Note that for E_S/N_0, the SNR is limited by P_U/P_I.

From (15) Concluded adding guard time consume part of signal power and so it can considered as an additive noise. [5] investigated optimal guard time according to channel multipath intensity profile.

V. Different guard time description

A.. CP mathematical model

As said, OFDM based on subcarriers orthogonality, so guardtime must serve this property[10, 11]. CP satisfies this condition by cyclically extension of block subcarriers. Suppose transmitted signal be as follow:

x[n]={x0[n] , x-1[x], ... xn-1[n]}. Adding CP convert it to *x[n]={xN-v[v], x-n-v+1[n],...xn-1[n], x0[n], x1[n],...,xn-1[n]}*.

The CP operation can be represented as a pre-multiplication by the matrix $P_T \in R^{N+v \times N}$ defined as:

$$P_T = \left[\frac{O_{(v \times N-1)} \,|\, I_v}{I_N} \right] \quad (16)$$

So the transmitted vector becomes

$$x[n] = P_T F_N^H d[n]. \quad (17)$$

Then the received signal (after passing through a linear channel) will be:

$$y[n] = H[n]x[n] + z[n]$$
$$= H[n]P_T F_N^H[n] + z[n] \quad (18)$$

Where $H[n] \in C^{N+2v \times N+v}$ is the channel matrix in familiar Toeplitz form and *z[n]* is additive noise. Removal of the cyclic prefix, which will be performed at the receiver, can be expressed as a multiplication by the matrix $P_R \in C^{N \times N+2v}$ defined as:

$$P_R \equiv \left[O_{(N \times v)} \middle| I_k \middle| O_{(N \times v)} \right] \quad (19)$$

The received signal after cyclic prefix removal and forward DFT becomes:

$$y'[n] = F_N P_R \left(H[n] P_T F_N^H d[n] + z[n] \right)$$
$$= F_N P_R H[n] P_T F_N^H d[n] + z'[n]. \quad (20)$$

By exploiting the following two facts:

- For the above choices of P_T and P_R, and for any Toeplitz H of appropriate size, the matrix $P_R H P_T$ will be circulate.
- Any circulate matrix is diagonalized by the DFT matrix [6].

It becomes apparent that the effect of OFDM transmission with CP extenuation is identical to multiplication of the original data sequence by a complex diagonal matrix. This diagonal matrix contains the complex fading coefficient for each frequency channel. Hence, only N parameters need to be estimated to the receiver for coherent detection.

Although CP extension serve orthogonality, but at multipath channel, destructive effect of CP subcarriers (multipath reflection effect) on tail of block consists of main signal, causes more ISI. To bypass this problem, adding TZ gaurdtime can be used.

B. TZ mathematical model

Multipath channel can be modeled by fir filter, so at a channel with v tap, adding v+1 zero subcarriers to block, omits ICI and causes more effect to reducing channel ISI than CP extension. The TZ operation can be represented as a pre-multiplication by the matrix $P_T \in R^{N+v \times N}$ defined as:

$$P_T = \left[\frac{O_{v*N}}{I_N} \right] \quad (21)$$

C. TZCP mathematical model

From previous methods, following results concluded:

- To prevent ICI and ISI effects, guard time must add to block.
- At fading channels, most efficient guard time is TZ.
- Applying TZ, cancel ISI in cost loss of orthogonality OFDM blocks.

We approached a new method to obtain better result from two previous methods by combination them.

By adding half guard time immediate after block by CP and remainder of guard time interval by TZ, TZCP extension obtained (figure 2). This combination reduces effect of guard time over main signal vs. CP, and less loss of orthogonality vs. TZ. Following section express simulation results and compare performance between different Guard times.

D. Simulations

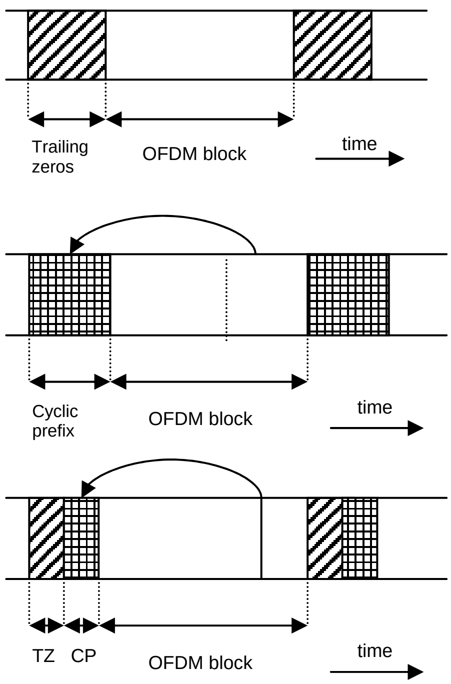

Figure 2- different guard time scheme

Simulations have been performed for a multicarrier system with IFFT size 1024, number of subcarrier 480, and 25% signal time interval added guard time, based on Monte Carlo simulations with each trial corresponding to a different realization of the channel modeled by fir filter with tap coefficients provided in [8, 9].

These taps simulated HF channel and a slow fading channel with f_d=1 *hz*, as occurs in the indoor environments. Input data stream arranged as dobinary data with gray coding configuration The four different methods for adding guard time used for this simulation were: TZ, CP, TZCP, OLA [10]. [10] presented a trailing zeroes guard time with equalization scheme based on the overlap-and add (OLA) block convolution algorithm and named it OLA-TZ. Results have been obtained for a DQPSK modulation.

From simulation result, concluded channel with low fading power, same HF channels, required less complexity and guard time to system and transmitted blocks, than slow fading channels (indoor environments), because the most efficient guard time for it is TZ (Fig. 3). To obtain similar BER, also, at slow fading channels, from Fig. 4 can be seen combination of CP and TZ method (TZCP) has the best result versus others.

Comparison between four methods of guard time insertion, demonstrated CP and OLA-TZ have closest performance to TZCP method. It can be inferred that performance gap between CP and TZCP is enlarged at higher fading power to AWGN power ratio.

E. Conclusion

This paper demonstrated performance of different guard type and presented TZCP. Simulations has shown TZCP guard time obtained better BER at fading channels. Also it can be concluded guard time at HF channel has less size for guard time and less complexity for transmitter versus slow fading channel to for equal BER.

Because CP guard time insertion is applied in OFDM systems nowadays, to consume lower power, TZCP can apply without any BER Penalty.

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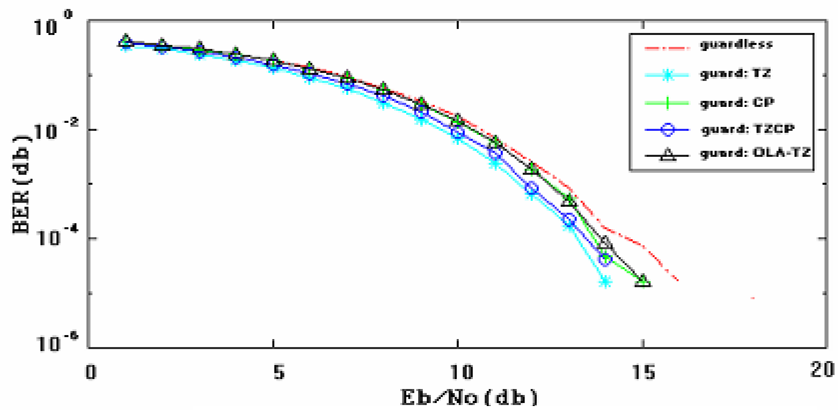

Figure 3 – BER comparison at HF channel

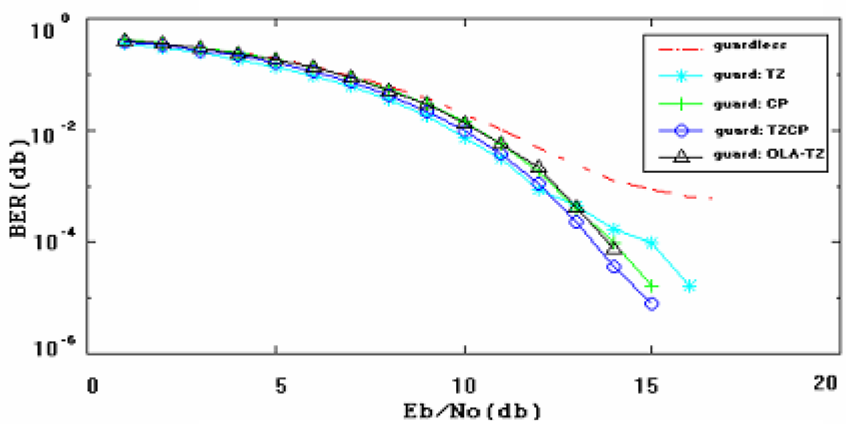

Figure 4 – BER comparison at fading channel