A Symbol Synchronization Algorithm for OFDM Systems

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***Abstract*—In the literature a number of methods are described for symbol synchronization of OFDM systems. Most of the references use the cyclic prefix of each symbol period to estimate the symbol timing. The insertion of this redundant information increases the symbol period and hence reduces the data transmission rate. Therefore a new algorithm is presented which is not based on the cyclic prefix. Such an algorithm can be used for systems with multi-path conditions where the cyclic prefix is used up by the multi-path delay.**

The algorithm derives an estimate of the symbol timing offset from pilot tones. A periodic pattern of four symbols is used for this purpose. The first pair of symbol periods is transmitted with identical polarity pilots and the next two symbols are modulated with the opposite polarity pilots. A correlation matrix is derived from the known data and is employed to detect the start of symbol period. Performance of the estimator exhibits low bias and variance.

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) has gained wide acceptance as a modulation scheme for transmission of broadband and especially multimedia information on wireless channels. In this technique the information is transmitted in many narrow parallel frequency bands [1]. This results in high bandwidth efficiency. Due to its higher data transmission rate OFDM has been adopted as the method of modulation for Digital Audio Broadcasting (DAB) and digital Video Broadcasting (DVB) [5]. It is also proposed in other systems like Asymmetric Digital Subscriber Loop (ADSL) and High Definition Television (HDTV) [5]. OFDM is used in these systems because of the cyclic extension of the symbol period that makes it comparatively robust against the multi-path propagation. However this results in loss of bandwidth efficiency.

In this article we investigate a new symbol synchronization algorithm for OFDM systems employing pilot tones. A correlation matrix is derived from training symbols and described in section 2. The correlation metric can be exploited using different methods to acquire the symbol start. In this case we employ zero crossing to adjust the receiver FFT window. In section 3 the algorithm is tested with pilot data and random 4QAM data. The residual timing offset can be compensated for using simple phase compensator which is discussed in section 4. The robustness of the algorithm is tested using averaging and variance in section 5. In the last section conclusions are drawn.

II. A NEW SYMBOL SYNCHRONIZATION ALGORITHM

The algorithm is based on a correlation function integrated over a symbol period. The structure of the signal for correlation is shown in Figure 1. The received samples $r(t)$ are correlated with delayed samples with delay of T symbol period. The start of the four symbol pilot pattern is represented by t_s. The width of the sliding window is N samples.

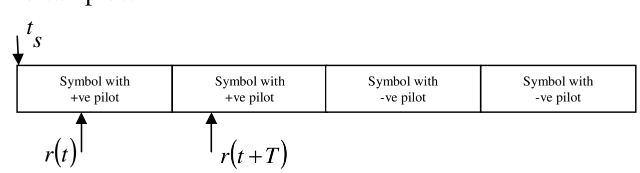

Figure 1 Structure of the signal for correlation.

The output of the sliding integrator with window size of N samples can be written as

$$S_{corr}(t) = \sum_{i=0}^{N-1} r\left(t + \frac{iT}{N}\right) r^*\left(t + \frac{iT}{N} + T\right) \tag{1}$$

where T is a symbol period and N is number of samples in a symbol period. The value of $S_{corr}(t)$ depends on the difference between t and t_s, which is the start of the positive polarity or the negative polarity pilot symbol pair.

A simulated result of the correlator is shown in Figure 2. When the first sample is at the start of the four symbol pilot pattern i.e. $t = t_s$, then each pair of samples in the sliding window have identical polarity, mathematically

$$r\left(t+\frac{iT}{N}\right)=r\left(t+\frac{iT}{N}+T\right) \quad \text{for} \quad 0<i<N-1 \tag{2}$$

Therefore all the terms in the sliding window add up and give a peak value of the correlation. From equation (1)

$$S_{corr}(t)=\sum_{i=0}^{N-1} r\left(t+\frac{iT}{N}\right) r^{*}\left(t+\frac{iT}{N}\right) \tag{3}$$

$$S_{corr}(t)=\sum_{i=0}^{N-1}\left|r\left(t+\frac{iT}{N}\right)\right|^{2} \tag{4}$$

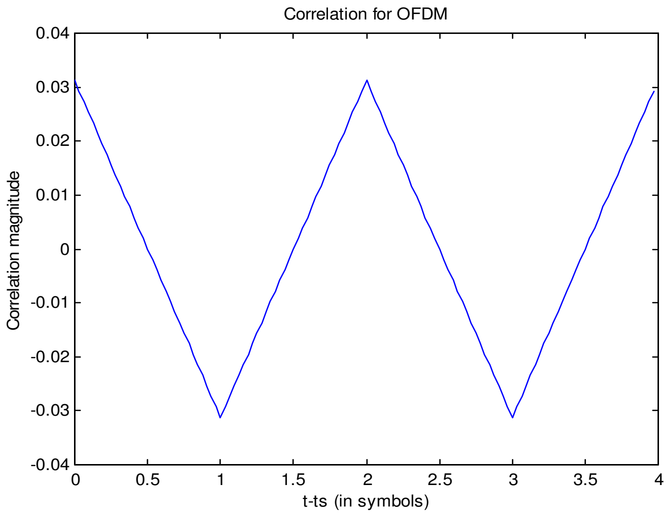

Figure 2 Correlation for the four symbol pilot pattern when $N=32$.

For $t=t_s+T/2$, the pairs of samples in the window are of opposite polarity i.e.

$$r\left(t+\frac{iT}{N}\right)=r\left(t+\frac{iT}{N}+T\right) \quad \text{for} \quad 0<i<N/2\text{-}1 \tag{5}$$

$$r\left(t+\frac{iT}{N}\right)=-r\left(t+\frac{iT}{N}+T\right) \quad \text{for} \quad N/2<i<N\text{-}1 \tag{6}$$

Thus the contribution to the summation from the first $N/2$ components is exactly cancelled by the contribution from the second $N/2$ components and correlation metric is zero.

The peak value of $S_{corr}(t)$ occurs at $t=t_s$, $t=t_s+2T$ and so on. The graph in Figure 2 shows the $S_{corr}(t)$ varying with the timing error $\tau_e=t-t_s$. The start of the symbol can be estimated using the positive or negative slope or the maximum value of the estimator. We use the negative going zero crossing to estimate the symbol start. The negative going zero crossing occurs when $t=t_s+T/2$, $t=t_s+5T/2$ and so on.

A. *Structure of the Algorithm*

The proposed estimator is used to estimate the symbol start. The receiver FFT window is adjusted using the negative going zero crossing of the correlation. A procedure is described in Figure 3, which aligns the FFT window with the received symbols. The correlation function is calculated from the received samples to align the FFT window with the start of symbol. The FFT window is moved forward or backward depending on the correlation output.

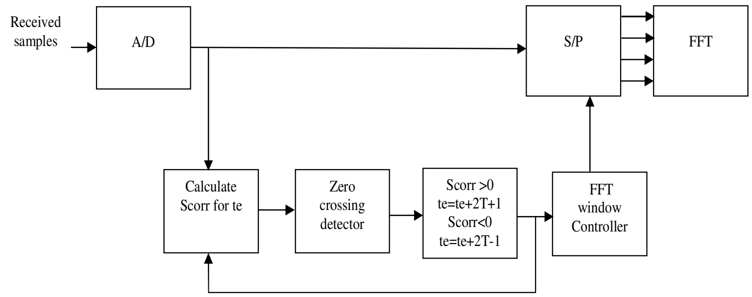

Figure 3 Block diagram of the symbol synchronization algorithm.

If the correlation is negative that shows the FFT window is late therefore it should be moved back for one sample. We can write for $t>t_s+T/2$

$$S_{corr}(t)=\sum_{i=0}^{N-1} r\left(t+\frac{iT}{N}\right) r^{*}\left(t+\frac{iT}{N}+T\right)<0 \tag{7}$$

On the other hand positive correlation shows FFT window should move forward i.e. for $t<t_s+T/2$

$$S_{corr}(t)=\sum_{i=0}^{N-1} r\left(t+\frac{iT}{N}\right) r^{*}\left(t+\frac{iT}{N}+T\right)>0 \tag{8}$$

Because the negative zero crossing of the estimator occurs after $2T$ delay, therefore the next negative slope is guessed after $2N$ samples delay.

III. SIMULATIONS OF THE ESTIMATOR

The estimator was tested for a system when only pilot data is modulated in a symbol period and for a situation where all the subcarriers are modulated with random 4QAM data.

A. *Symbols with Pilot Data*

For this simulation only pilot data is modulated and the rest of the subcarriers are set to zero. The estimator is based on the negative going zero crossing of the correlation and the next negative zero crossing occurs after $2T$ delay. The initial estimated timing error may occur on the negative going or positive going zero crossing. For the first instance we consider that the initial timing estimate is somewhere on the negative going slope. The estimator reduces the timing error to one sample as shown in Figure 4. It is shown that the alignment of the FFT window with the symbol start is very fast.

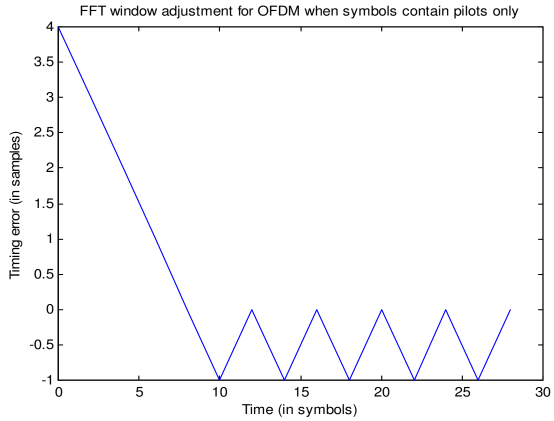

Figure 4 Timing error versus symbol timing when symbols are transmitted with pilots where $N = 32$, $m = 32$.

The estimator takes a longer time to acquire the symbol start when the initial timing estimate is on the positive going slope. The estimator moves the FFT window until the zero crossing on the negative slope is reached. When the timing offset is on the positive slope, the symbol acquisition time is longer in this case but the residual time is identical to the previous case.

B. Symbols with 4QAM Random Data

Now we test the algorithm using 4QAM random data transmission. In this case interference caused by the random data affects the performance of the estimator as shown in Figure 5. The estimator cannot keep the timing error to one sample period. The algorithm is based on the correlation integrated over a symbol period. Since only one subcarrier is modulated with known data, the ratio of correlated components to the uncorrelated components in a symbol period is decreased. Therefore the estimator has a large variance in this case and is unable to track the zero crossing.

In OFDM a large value of N is required to mitigate the multi-path effects. For our case if the value of N increases the ratio of the independent components to the correlated components in a symbol period increases. Therefore the amplitude of the jitter in the estimate increases.

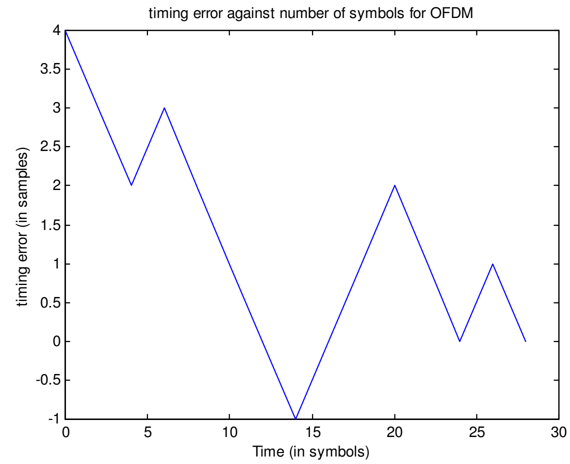

Figure 5 Timing error versus symbol timing when symbols are transmitted with 4QAM where $N = 32$, $m = 32$.

To mitigate the jitter in the estimator two methods can be adopted. Firstly the pilot data can be modulated with a higher magnitude than the transmitted data. Since the component of correlation due to the pilots is increased relative to the component due to the random data, the relative power of the unwanted component is reduced. This method requires more power to transmit higher amplitude pilot tones.

Secondly averaging can be performed over the four symbol pilot pattern to increase the reliability of the estimator. In this case the correlated components add up and components due to random data do not. If any of the methods are implemented, the timing error can be reduced to one sample interval. The residual timing error can be partially compensated using a frequency domain equalizer.

IV. COMPENSATION OF RESIDUAL TIMING ERROR

Once the timing error is tracked to within a sampling period, the residual timing offset can be partially compensated at the output of the FFT. This method can be performed more easily than correcting the sampling time instant in the analog domain. The magnitude of phase rotations in the received data caused by the residual timing error is directly proportional to the subcarrier frequency and the symbol timing offset. To correct the phase rotations a phase compensator is used at the output of the receiver FFT. The mathematical form of the compensator is

$$\phi_k = \exp\left(\frac{-j2\pi k\Delta t}{T}\right) \quad (9)$$

where k is subcarrier index, Δt is timing offset and T is symbol period.

Simulations were carried out to show the effect of residual timing offset on the received constellation when the timing error has been reduced to one sample time. The timing error causes spread and phase rotations in the data as shown in Figure 6 (a). The output of the phase compensator is displayed in Figure 6 (b). The graph shows that there is still some smearing of constellation points after the phase correction. This is because no cyclic prefix is used and as a result ISI and ICI are still present after the phase rotation is corrected.

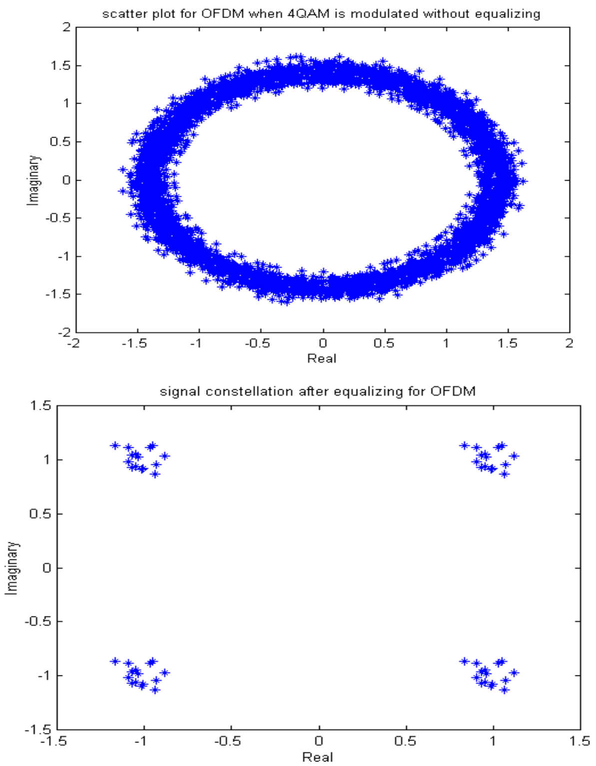

Figure 6 Received signal constellation (a) without and (b) after phase compensation for OFDM, $m = 32$ and $N = 256$.

V. PERFORMANCE OF THE ESTIMATOR

In this section we discuss performance of the estimator employing random 4QAM data. When the algorithm is implemented using the random 4QAM modulation scheme, the ICI from the subcarriers affects the output of the estimator. This introduces jitter in the correlation as shown in Figure 7(a). Accuracy of the estimator can be increased by averaging over number of symbols. The output of the correlation matrix is shown in Figure 7(b) when averaging is done for 100 thousand four symbol pilot patterns. It can be seen that performance of the estimator is considerably improved when long term averaging is performed. The estimator can reasonably work well in hostile environment provided averaging filter or noise whitening procedure is in place [4].

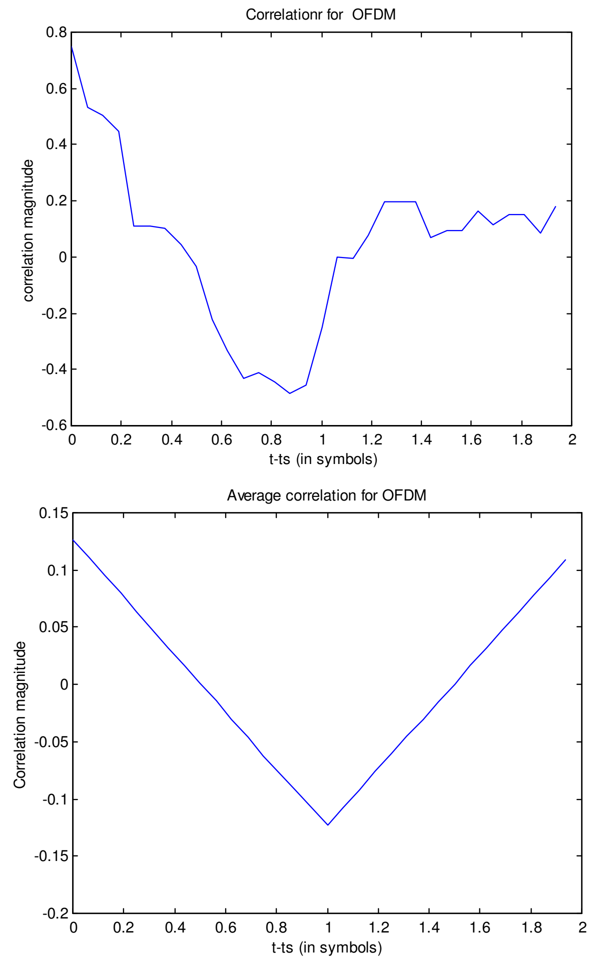

Figure 7 (a) Correlator output without averaging (b) after averaging of four symbol pilot pattern.

A. Variance of the Estimator

Now we discuss a second parameter, variance, to measure the quality of the estimator. The variance is a measure of the performance of the algorithm in the presence of interference in the timing estimate [3]. Variance of the estimator has been simulated and is shown in Figure 8 for $N = 16$.

The value of variance is very small and the dependency of variance on the timing offset is negligible. When the timing offset is zero i.e. $t = t_s$, the timing error does not produce the ICI therefore the variance has its smallest value. With increasing time offset the variance increases with ICI. When the timing offset is $t = t_s + T$, which is the start of the negative polarity pilot symbol pair, the timing offset does not create ICI and the variance decreases. Thus the variance of the timing estimate increases with increase in the timing offset although the change in variance is almost negligible.

For an application where a large value of N is a requirement, the number of uncorrelated components would relatively increase. When the timing offset is zero, the variance would be even smaller than the previous case. The power of ICI would comparatively decrease with increasing time offset because the large value of N produces a large signal to interference ratio

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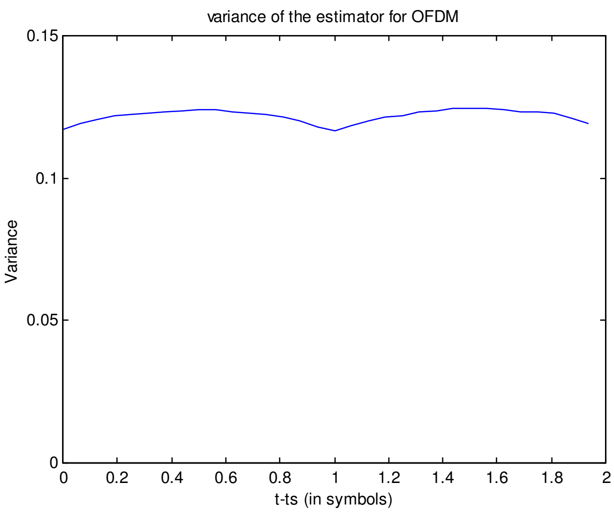

Figure 8 Variance of the estimator as a function of timing offset

VI. DISCUSSIONS

A new symbol synchronization algorithm is presented for OFDM. The estimator is based on the correlation of the received samples when a pattern of four symbols is periodically transmitted with pilot tones. Positive and negative slopes are derived from the correlation. The symbol timing recovery is performed using the negative slope, however the results would be the similar for the positive slope. The symbol acquisition time is very fast and the timing error is tracked to one sample interval. The only limitation of the estimator is the longer symbol acquisition time when the initial timing estimate is on the positive slope.

Another advantage is reduced receiver complexity because no fine symbol synchronization stage is required. A phase compensator is implemented at the outputs of FFT to minimize the residual timing error. The performance of the estimator is tested by calculating the average value and the variance. The long term averaging improves the estimator performance. The change in variance with the timing error is very small; however, it slightly increases with increasing the timing error.

VII. REFERENCES

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