Performance of a Turbo Coded LEO DS-CDMA System in Uncorrelated and Correlated KA-Band Channels

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Abstract— **In this paper, the performance of a turbo coded Ka-band code-division multiple access (CDMA) based land mobile satellite system is presented. An analytical bound based on the union bounding technique is derived for the two state channel model and shown to be useful in estimating the performance of the system. Simulation results of the system are presented considering both the correlated and the uncorrelated channel. Other cell interference which is of prime interest in satellite systems is also considered.**

Index Terms— ***DS-CDMA, Turbo codes, LMSS, fading channel, bound.***

I. INTRODUCTION

A major goal for the next generation communications is to provide basic communication services anywhere and anytime at an affordable cost with efficiency. Satellite based networks and terrestrial networks are set to complement each other to achieve this goal with the former suitable for rural, maritime and aeronautical environments where there are few users and the latter is suitable for urban environments where there many users and obstructions. The satellite personal communication networks are also considered to provide an umbrella cell structure for terrestrial networks and assist in minimizing call blocking or dropping in the latter. Also the improvement in satellite communications has seen satellite networks being considered to provide completely ubiquitous and seamless services to the mobile user.

The overcrowding of the lower frequency bands namely; L-, S-, C-, and Ku-bands leads to the Ka-band being the most suitable band for the next generation satellite communications [1]. The Ka-band is also suitable for broadband multimedia services due to its wide bandwidth that allows the use of high data rate traffic. However, the Ka-band suffers from a hostile propagation environment [2]. The propagation environment is mainly affected by shadowing and faster multipath fading [3]. The atmospheric and tropospheric effects contribute severe adverse effects on this band as in the lower bands [1]. Doppler effects are also severe in this band.

To minimize the effects of the bad propagation environment, high coding gain channel codes such as turbo codes and Low Density Parity Check Codes (LDPC) are used. These codes can be used to further minimize power usage in such power constrained systems. Direct Sequence Code Division Multiple Access (DS-CDMA) is considered for the next generation wireless network and it is suitable for satellite systems. Other cell interference is a major concern in satellite DS-CDMA networks [9].

In this paper, we present the performance of a turbo coded DS-CDMA Low Earth Orbit (LEO) satellite system operating in the Ka-band. We provide simulation and analytic results of this system in different conditions that are defined by the communication channel statistics and the satellite system. Section II presents the system model and in Section III, the bound is derived. In Section IV, we present the correlated fading concept and results are presented in Section 5. Section VI concludes the paper.

II. SYSTEM MODEL

A. The Land Mobile Satellite Channel (LMSC)

We consider a two state statistical channel model that was introduced in [4] whereby the channel is divided into two states; the good state and the bad state. A time share determines the severity of the shadowing and it is a function of the user location. The probability density function (pdf) of the general two state channel model is given by [4];

$$p(\rho) = \left(1 - A\right) p_{Rice}(\rho) + A\int_0^\infty p_{Rayl}(\rho \mid \rho_o) p_{LN}(\rho_o) d\rho_o \qquad (1)$$

where, A is the time-share of shadowing and the integral expression signifies total probability. $p_{Rice}(.)$ represents the

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Rice distribution, $p_{Rayl}(.)$ is the Rayleigh distribution and $p_{LN}(.)$ is the Lognormal distribution. ρ is the received signal envelop and ρ_o is the short term mean power of the multipath fading.

The pdf of ρ_o is,

$$p_{LN}(\rho_o) = \frac{10}{\sqrt{2\pi}\sigma\rho_o ln10} exp\left[\frac{-\left(10log\rho_o - \mu\right)^2}{2\sigma^2}\right] \tag{2}$$

where μ and σ are the mean and the standard deviation in dB of the associated normal variate respectively. By utilizing the parameters; A, μ , σ and the Rice Factor , the two state model can be fully described. The parameter A is the most important in the LMSC model [4]. With $A=1$, the channel is Rayleigh – Lognormal, representing the presence of shadowing, and with $A=0$, the channel is Rician, representing a clear line of sight.

B. Turbo Codes

Turbo coding involves the parallel concatenation of two constituent recursive systematic convolutional (RSC) encoders separated by an interleaver [5]. The information bits are encoded using one RSC encoder and they are also interleaved before being encoded using a second RSC encoder. The transmitted symbol is the information bit followed by the two parity bits from the constituent codes. The turbo decoder consists of two soft input soft output decoders that share information iteratively [5].

C. Receiver Module

We consider a multi – satellite system (satellite diversity system). There are L LEO satellites serving as base stations with S spot beams and transparent transponders. K mobile terminals on the earth surface communicate with the satellites. The received signal is given by [6],

$$\begin{aligned} r(t) = n(t) + \sqrt{2P_t}\sum_{k=1}^{K}\sum_{l=1}^{L}\rho_l^k b^k\left(t - \tau_l^k\right)c^k\left(t - \tau_l^k\right)cos\left(\omega_c t + \varphi_l^k\right) \\ + \sqrt{2P_t}\sum_{k=K+1}^{SK}\sum_{l=1}^{L}\rho_l^k\beta_l^k b^k\left(t - \tau_l^k\right)c^k\left(t - \tau_l^k\right)cos\left(\omega_c t + \varphi_l^k\right) \end{aligned} \tag{3}$$

where $n(t)$ is the additive white Gaussian noise (AWGN) with two – sided power spectral density $No/2$, P_t is the transmitter power and ρ_l^k is fading parameter at the lth path and kth user respectively with pdf given by (1). $c^k\left(t\right)$ is the spreading sequence of the kth user generated at a rate $1/Tc$, and $b^k\left(t\right)$ is the binary information data for the kth user generated at a rate $1/Tb$. Tc and Tb are the chip duration and bit duration respectively. Therefore, the processing gain, Nc, is given by $Nc=Tb/Tc$. τ_l^k and φ_l^k are the time delay and carrier phase respectively. β_l^k is the spot beam isolation coefficient and it represent the effect of spot beam antenna patterns. It also accounts for the multiple access interference (MAI) that the lth satellite illuminating user k imposes on the desired user. These coefficients are assumed to be the same for all the satellites, i.e. $\beta_l^k = \beta^k$ and also $\beta^1 = 1$ for the user of interest [6, and references inside].

Assuming a single path from each satellite, using a coherent Binary Phase Shift Keying (BPSK) receiver, and assuming that the receiver locks on each path, the Maximum Ratio Combined (MRC) received signal from an L – finger Rake receiver, for a specific user of interest, user $k = 1$, is;

$$\begin{aligned} U^1 &= \sum_{l=1}^{L}\left\{\int_{\tau_l^1}^{T_b+\tau_l^1}\sqrt{\frac{2}{T_b}}r(t)\rho_l^1 c^1\left(t - \tau_l^1\right).cos\left(\omega_c t + \varphi_l^1\right)dt\right\} \\ &= \sum_{l=1}^{L}\left\{X_l + V_l + \eta_l\right\} \end{aligned} \tag{4}$$

where,

$$X_l = \sqrt{P_t T}\left\{\rho_l^1\right\}^2 b_0^1 \tag{5}$$

$$V_l = \sqrt{\frac{P_t}{T_b}}\rho_l^1\sum_{k=2}^{SK}\sum_{l=1}^{L}\rho_l^k\beta^k\left\{\begin{aligned} &b_{-1}^k R_{1k}\left(\tau_l^k - \tau_l^1\right) \\ &+b_0^k\hat{R}_{1k}\left(\tau_l^k - \tau_l^1\right)\end{aligned}\right\}cos\left(\varphi_l^k - \varphi_l^1\right) \tag{6}$$

$$\eta_l = \int_0^{Tb}\sqrt{\frac{2}{Tb}}\rho_l^1 n(t)c_l^1(t - \tau_l^1)cos(\omega c t + \varphi_l^1)dt \tag{7}$$

X_l,, V_l and η_l represent the desired user's signal, the MAI and the AWGN components. b_{-1}^k and b_0^k represent the previous binary information data bit and the current information bit respectively for the kth user. $R_{1k}\left(\tau\right)$ and $\hat{R}_{1k}\left(\tau\right)$ are cross correlation coefficients of the spreading codes and are given by,

$$R_{1k}(\tau) = \int_0^{\tau}c^1\left(t - \tau\right)c^k\left(t\right)dt$$

$$\hat{R}_{1k}(\tau) = \int_{\tau}^{T_b}c^1\left(t - \tau\right)c^k\left(t\right)dt$$

III. ANALYSIS

Assume; $\boldsymbol{x_o}$, to be the transmitted codeword over a fading channel, $\boldsymbol{x_j}$, to be the received codeword and that the receiver can accurately estimate the channel, i.e. the fast fading $\boldsymbol{\rho}$ is perfectly known at the receiver [2]. The probability that the decoder incorrectly decodes $\boldsymbol{x_o}$ into a codeword $\hat{\boldsymbol{x}}_{\boldsymbol{o}}$ is known as the Pair - wise Error Probability (PEP). To evaluate the PEP, perfect channel interleaving is assumed which results in an independent Rician or Rayleigh fading for each code symbol [2]. Only the weight of the incorrect codeword matters. The conditional PEP conditioned on a is given by [2 and references inside],

$$P_2(\boldsymbol{x_o}, \hat{\boldsymbol{x}}_{\boldsymbol{o}} \mid \boldsymbol{\rho}) = Q\left(\sqrt{2R\frac{E_b}{N_0}\sum_{j=1}^{d}\rho_{ij}^2}\right) \tag{8}$$

where R is the code rate, d is the maximum number of differing bit positions between $\boldsymbol{x_o}$ and $\hat{\boldsymbol{x}}_{\boldsymbol{o}}$, ij is the index of the differing bit positions and $Q(z)$ is given by the Craig's formula;

$$Q(z) = \frac{1}{\pi}\int_0^{\pi/2} e^{-z^2/(2\sin^2\varphi)}d\varphi \tag{9}$$

The Gaussian approximation is used to evaluate the noise variance of V_l and the output signal-to-noise ratio (SNR) of the MRC receiver is written as,

$$\gamma = \sum_{l=1}^{L}\frac{\langle X_l \rangle^2}{2\sigma_l^2} \tag{10}$$

where σ_l^2 is the total variance of the MAI and the AWGN components for the lth path and $\langle X_l \rangle$ is the mean of the received signal given by;

$$\langle X_l \rangle = \sqrt{P_t T}\left\{\rho_l^1\right\}^2 \tag{11}$$

The PEP is obtained by averaging over the channel fading gain, ρ,

$$P_2(d) = P_2(\boldsymbol{x_o}, \hat{\boldsymbol{x}}_{\boldsymbol{o}}) = (1 - A)\int_{\{\rho\}} p_{Rice}(\boldsymbol{\rho})Q(s)\,d\{\boldsymbol{\rho}\} + A\int_{\{\rho\}}\int_0^\infty p_{Raly}(\boldsymbol{\rho} \mid \boldsymbol{\rho_0})p_{LN}(\boldsymbol{\rho_0})\,d\rho_0 Q(s)\,d\{\boldsymbol{\rho}\} \tag{12}$$

where

$$s^2 = \frac{LE_s}{\left(\frac{E_s y\left[(K - 1) + \sum_{k=K+1}^{SK}\beta^k\right]}{3N_c} + \frac{N_0}{2}\right)} \tag{13}$$

where Es is the symbol energy and in this equation, it is assumed that all signals from the satellites undergo the same propagation channel effects. y is the second moment of the fading variable, $y = E\left[\left\{\rho_m^k\right\}^2\right]$. In the case of normalized Rician or Rayleigh, y becomes 1. For the Rayleigh – Lognormal case,

$$\begin{aligned} y &= E\left[r^2\right]E\left[r_o^2\right] \\ &= 2\sigma_R^2 \exp\left(2h\mu + 2h^2\sigma_L^2\right) \end{aligned} \tag{14}$$

The bit error rate (BER) of the turbo code can be obtained by assuming maximum likelihood decoding and uniform interleaving. It is upper bounded by,

$$P_b \le \sum_{d \ge dmin} A_d P_2(d) \tag{15}$$

A_d is the error coefficient and is the average number of bit errors caused by the transition between the all – zero codeword and codewords of weight d. d_{min} is the minimum Hamming distance of the code. A_d is calculated in [7] and $P_2(d)$ is evaluated numerically using Gaussian Quadratures; the Gauss – Laguerre and the Gauss – Legendre quadratures. The Gauss – Laguerre quadrature is a Gaussian quadrature over the interval [0 - ∞] and is used to evaluate those integrals that are in this interval. The Gauss – Legendre quadrature is a Gaussian quadrature over a finite interval [a – b] and is used to evaluate integrals falling in this interval. The general representation of structure of Gaussian quadratures is;

$$\int_a^b f(x)dx = \sum_{i=1}^{n} w(x_i)f(x_i) \tag{16}$$

where [a, b] is the interval of integration, x_i are the abscissas or points of integration that are associated with the zeros of orthogonal polynomials, $w(.)$ is the weighting function and is related to the orthogonal polynomials [8].

IV. CORRELATED FADING

We use the same channel model as before and we evaluate the performance of the system in a correlated multipath scenario, considering the Doppler Effect. The Jakes power spectral density, $S_a(f)$, of the multipath channel process is given by;

$$S_a(f) = \begin{cases} \dfrac{2\sigma_a^2}{\pi\sqrt{f_D^2 - f^2}} & |f| < f_D \\ 0 & \text{elsewhere} \end{cases} \tag{17}$$

where $2\sigma_a^2$ is the diffuse power in the diffuse process and f is the frequency variable. f_D is the maximum Doppler frequency and is given by;

$$f_D = \frac{vf_c}{c} \tag{18}$$

where v, f_c, and c are the mobile receiver's velocity, carrier frequency and propagation speed (speed of light) respectively. The power spectral density and autocorrelation function are Fourier transform pairs and therefore the autocorrelation function, $R_{aa}(t)$, is given by;

$$R_{aa}(t) = 2\sigma_a^2 J_o(2\pi f_D t) \tag{19}$$

where t is a time variable and $J_o(.)$ is the zero order Bessel function of the first kind. The Jakes spectrum is commonly used in modeling land mobile channels. In our work we vary the normalized Doppler frequency, *fDT*, that is, the ratio of the Doppler frequency to the sampling frequency.

V. RESULTS

The turbo code considered consists of two $(7,5)_8$ constituent RSC codes separated by a pseudorandom interleaver. The Rice factor is set to 12dB and μ = -10dB. The number of iterations is set at 10 and the frame length is set at 100 unless stated. The data rate is 64kbps.

In Fig 1, a comparison of the system performance in uncorrelated and correlated fading situations is shown for a normalized Doppler frequency of 0.1 and frame length of 1000. It is seen that the correlation degrades the performance of the code. The waterfall region is shifted to a higher Eb/No and the error floor region is raised. In Fig 2, we present the bound compared with simulation results for a system with 10 users in the spot beam of interest and we take the overall interference from users in other spot beams to be half of the interference from the users of the spot beam of interest. The usefulness of the derived bound is clearly shown as it is tight to simulation results for an uncorrelated channel. In Fig.3, it is seen that performance is improved with the use of a channel interleaver. The interleaving depth, N, that is considered is 5 and 10 (it is the number of encoded frames). The gain achieved with increase in N is reduced as N increases. We used the pseudorandom interleaver and during our simulation we realized that there is no much difference between the performance of the random interleaver and the block interleaver.

In Fig.4, the degradation in system performance is shown as the time share of shadowing, *A*, is increased. It can be seen that the code performs badly in heavily shadowed conditions. In Fig.5, the results show the performance of the system as σ changes and Fig.6 presents the analytic results of the system when the other cell interference is considered.

VI. CONCLUSION

In this paper we have shown the performance of a turbo coded satellite system utilizing a two state satellite channel model. We also derived and evaluated the union bound to evaluate the performance of turbo codes in satellite systems using the two state channel models. Numerical results are presented and the derived bound is shown to be tight.

The results also show that the performance of the code is degraded in the presence of correlation and a small interleaver can be used to improve it. The overall results reveal that the system performs well in open areas where there is a significant presence of a line of sight signal.

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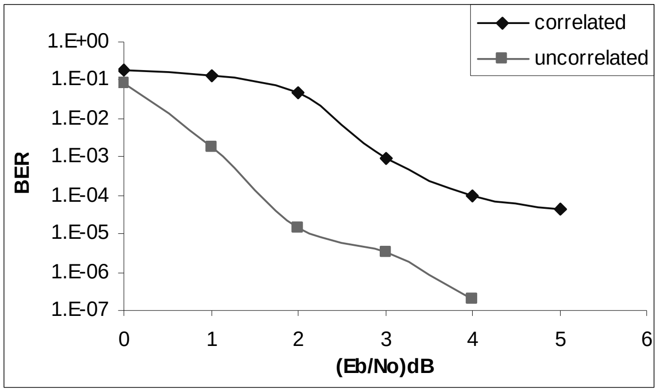

Figure1. Performance comparison in correlated and uncorrelated fading. [σ = 1dB, *A*=0.1, fame length = 1000, *fDT*=0.1.]

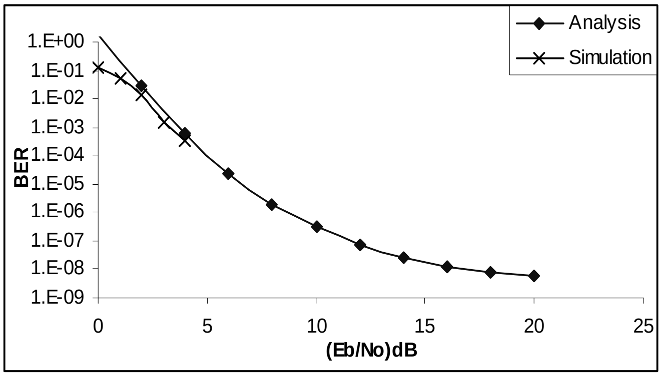

Figure2. Performance comparison of simulation and analysis. Frame length = 100. [*Nc*=127, *K*=10, frame length = 100]

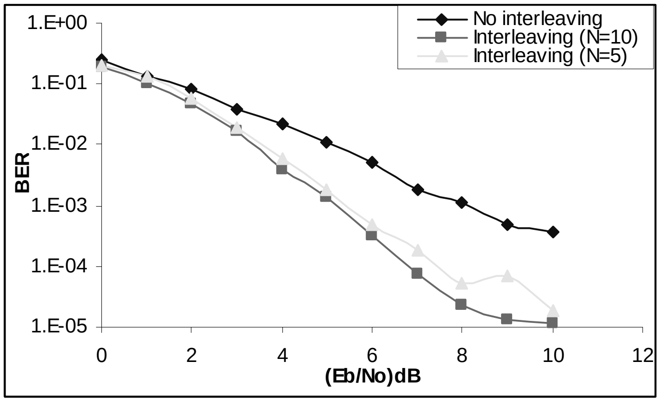

Figure3. The effect of interleaving. [*fDT*=0.01, frame length = 100]

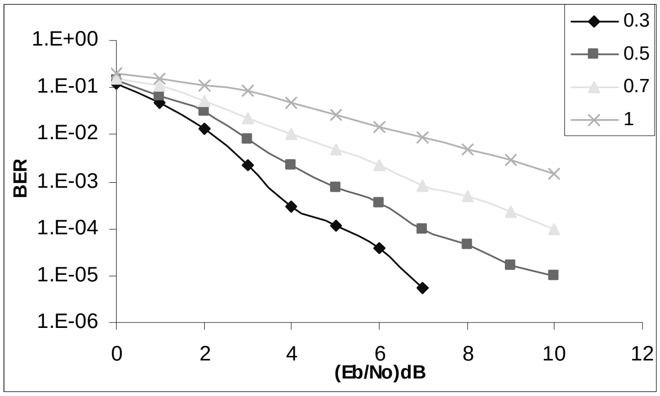

Figure 4. Performance of the system as the time share of shadowing changes. [σ = 1dB]

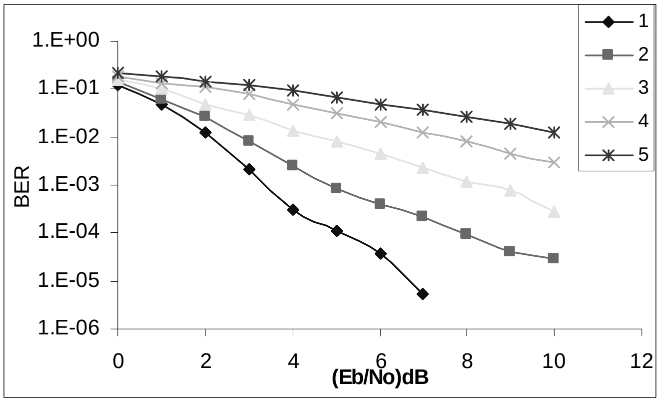

Figure 5. Performance as σ changes. [*A*=0.3, 1 user]

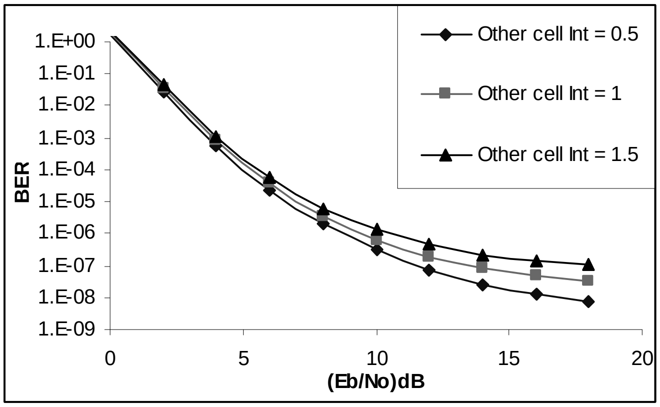

Figure 6. Effect of other cell interference.