Reliability of PDA based Target Tracking in Clutter

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Abstract – *Data association target tracking algorithms assume knowledge of target detection probability and sensor measurement noise. They are unknown and/or variable. Target signal is unknown due to unknown and time-varying target radar cross section as well as multi-path signal attenuation and variable propagation signal losses. Noise power is unknown due to the presence of unknown clutter signal whose residual power will increase the detection noise level, electronic counter-measures will also increase the noise power.*

The probability of target detection and the sensor measurement noise are affected by the (unknown) signal to noise ratio. Sensor measurement noise is also affected by proximity of large clutter reflectors due to the finite resolution capabilities of the sensor. This paper is a study of the effects of unknown probability of detection and sensor noise to the false track discrimination capabilities of IPDA and IMM-PDA target tracking algorithms.

Keywords: Target tracking, data association, reliability, PDA, IPDA, IMM-PDA, Markov Chain Two, false track discrimination.

1 Introduction

Data association algorithms deal with situations where there are measurements of uncertain origin. In many radar and sonar applications measurements (detections) originate from both targets and non-targets, i.e from various objects such as terrain, clouds etc., as well as from thermal noise. Unwanted measurements are usually referred to as clutter. In addition, the target measurements are present at each measurement scan with only a certain probability of detection. In a multi target situation, the measurements may have originated from one of several targets and the number of such measurements is generally unknown.

Automatic track initiation and termination under such conditions requires some knowledge about target existence. If a track follows a target, we shall call it a true track otherwise we shall call it a false track. Practical target tracking requires a mechanism to distinguish between true and false tracks, without a-priori knowledge of target's existence and trajectory. When a track is considered to be a true track, it is confirmed and presented to the operator or next processing level. When a track is considered to be a false track, it is terminated. We call this process "False Track Discrimination". We consider this to be a most important capability of a filter used for target tracking in clutter.

There is a variety of algorithms for target tracking in clutter. Algorithms can address single or multiple target tracking. They can be single scan algorithms, if they compress measurement history into one track state estimate, or multi scan algorithms, if they keep measurement history separate over last number of scans. For reasons of clarity and simplicity, the reliability study presented below is applied to two single target single scan algorithms based on Probabilistic Data Association (PDA) [**?**], namely Integrated PDA (IPDA) [**?**] and Interacting Multiple Model PDA (IMM-PDA) [**?**]. PDA algorithm addresses target trajectory state estimation in clutter only. IPDA and IMM-PDA extend PDA to include a measure of track quality; IPDA and IMM-PDA recursively calculate the probability of target existence and the probability of target detectability, respectively. Track quality measure is used for false track discrimination.

Track quality measures of IPDA and IMM-PDA ultimately depend on:

- The probability of target detection, P_D,
- Sensor measurement noise variance, R,
- Target trajectory model, and
- Clutter measurement density.

In this paper we assume that target trajectory model is not an issue; target maneuver modelling and state estimation is the topic of numerous contributions [**?**]. Variations in clutter measurement density is also ignored as an issue; the study presented in this paper assumes a heavy and non-uniform clutter situation, which does not change from experiment to experiment. The reason for this is that we are interested in a difficult clutter situation; if the clutter is less dense it will only improve target tracking performance.

The probability of target detection, P_D, and the sensor measurement noise variance, R, are assumed known. In most cases this is incorrect. One reason is that sensor specifications will drift with time. The more important reason is that they will depend on the signal to noise ratio. Neither signal nor noise are known in advance. The signal power changes because

- target radar cross section (RCS) will change significantly from scan to scan (Swerling models) [**?**],

- signal may get significantly attenuated due to multipath fading [?] and environmental losses [?],
- target may be obscured by an obstacle.

The noise power will change because

- a portion of clutter signal will filter through the moving target indicator (MTI) [?] in fact adding to the noise,
- electronic counter measures (ECM) often result in increased noise.

In addition, the measurement error covariance will change when the target measurement is merged with a clutter measurement, due to finite sensor resolution; IPDA-FR [?] may be used when this effect is significant.

This paper is a study on the reliability of false track discrimination capabilities of IPDA and IMM-PDA, when the probability of detection P_D and the sensor measurement noise R are unknown. It also attempts to find the best strategy to increase the target tracking reliability in such an environment. The authors are not aware of another publication with similar intentions. This is a draft paper and the final results are yet to be included in the paper, if it is accepted for presentation at the conference.

The rest of the paper is organized as follows: IPDA and IMM-PDA are briefly presented in Section 2. False track discrimination reliability study is presented in Section 3, followed with the concluding remarks in Section 4.

2 IPDA and IMM-PDA

In this Section we present a brief description of IPDA [?] and IMM-PDA [?]. Both IPDA and IMM-PDA are based on probabilistic data association (PDA) [?].

2.1 PDA

PDA approximates a-posteriori state probability density function (pdf) with a single Gaussian pdf with the same mean and covariance as the original pdf. Denote with z_k the set of $m_k \geq 0$ validated measurements at scan k, with V_k the volume of the selection window, and P_D and P_G the probabilities of target detection and selection, respectively. Let $\hat{x}_{k|k}$ and $\hat{x}_{k|k-1}$ denote state estimate and prediction mean respectively, with corresponding error covariances $P_{k|k}$ and $P_{k|k-1}$. Then, denoting with $\hat{x}_{k|k}$ and $P_{k|k}$ state estimate mean and variance respectively, and with $\hat{x}_{k|k,i}$ and $P_{k|k,i}$ state estimate mean and variance respectively given that measurement $z_{k,i} \in z_k$ is the correct measurement ($i = 0$ indicates that none of the selected measurement is the correct one),

$$\hat{x}_{k|k} = \sum_{i=0}^{m_k} \beta_{k,i} \hat{x}_{k,i|k} \tag{1}$$

$$\hat{P}_{k|k} = \sum_{i=0}^{m_k} \beta_{k,i} \left(\hat{P}_{k,i|k} + \hat{x}_{k,i|k} \hat{x}_{k,i|k}^T \right) - \hat{x}_{k|k} \hat{x}_{k|k}^T \tag{2}$$

$$\beta_{k,0} = \frac{1 - P_D P_G}{1 - \delta_k} \tag{3}$$

$$\beta_{k,i} = \frac{P_D P_G V_k p_i}{\hat{m}_k (1 - \delta_k)} \tag{4}$$

$$\delta_k = \begin{cases} P_D P_G; & m_k = 0 \\ P_D P_G \left(1 - \frac{V_k}{\hat{m}_k} \sum_{i=1}^{m_k} p_i\right); & m_k > 0 \end{cases} \tag{5}$$

with p_i denoting pdf of measurement $z_{k,i}$ and with $\hat{m}_k$ denoting the expected number of selected clutter measurements, for PDA and IMM-PDA $\hat{m}_k = m_k$. Likelihood ratio of the measurement set z_k is equal to $(1 - \delta_k)$ [?].

PDA does not include formulae to update a measure of track quality, which would enable false track discrimination. Both IPDA [?] and IMM-PDA [?] extend the PDA approach with recursive calculation of the track quality measure.

2.2 IMM-PDA

IMM-PDA [?] uses two PDA filters. One assumes $P_D = 0$, while the other uses nominal P_D. These two filters are combined using Interacting Multiple Model (IMM) scheme [?], where each PDA filter becomes one model. In this subsection superscripts 0 and 1 denote the $P_D = 0$ and $P_D = P_D$ model respectively. The a-posteriori probability of the models are denoted with $\mu_{k|k}^i, i = 0, 1$. The probability of target detectability is defined as $\mu_{k|k}^1$, and is used as the track quality measure. IMM-PDA track quality measure update consists of the following steps:

PDA update: When the measurement set z_k becomes available, both PDA models are updated using (1 - 5), producing $\hat{x}_{k|k}^i$ and $P_{k|k}^i, i = 0, 1$.

Model probability update: Denote with λ_k^i likelihood ratio of each model, then

$$\lambda_k^i = 1 - \delta_k^i \tag{6}$$

$$\mu_{k|k}^i = \frac{\mu_{k|k-1}^i \lambda_k^i}{\sum_{j=0}^{1} \mu_{k|k-1}^j \lambda_k^j} \tag{7}$$

where δ_k^i for each model is given with (5).

IMM mixing Denote with $M_k^i, i = 0, 1$ the event that model i is correct at time k, and denote the Markov model transition probabilities with

$$\pi_{ij} = P\{M_k^j | M_{k-1}^i\} \tag{8}$$

then

$$\mu_{k+1|k}^i = \sum_{j=0}^{1} \pi_{ji} \mu_{k|k}^j \tag{9}$$

$$\mu_{k|k+1}^{i,j} = \frac{\pi_{ij} \mu_{k|k}^i}{\mu_{k+1|k}^j} \tag{10}$$

$$\hat{x}_{k|k}^{i,m} = \sum_{i=0}^{1} \mu_{k|k+1}^{j,i} \hat{x}_{k|k}^{j} \quad (11)$$

$$\hat{P}_{k|k}^{i,m} = \sum_{i=0}^{1} \mu_{k|k+1}^{j,i} \big(\hat{P}_{k|k}^{j} + \hat{x}_{k|k}^{j} \hat{x}_{k|k}^{j\,T} \big) - \hat{x}_{k|k}^{i,m} \hat{x}_{k|k}^{i,mT} \quad (12)$$

IMM prediction Each PDA model predicts the state from k to $k+1$ using $\hat{x}_{k|k}^{i,m}$ and $P_{k|k}^{i,m}, i = 0, 1$.

2.3 IPDA

IPDA [**?**] extends PDA with the concept of target existence. A track is false if it does not follow a target's trajectory, i.e except when accidentally crossing a target's trajectory it is being updated by clutter measurements only. Alternatively, a track may be true if it is being updated by a target's measurements (and perhaps clutter measurements as well) over a period of time. The target existence event is, therefore, synonymous with the event that the track is a true track. The probability of target existence is updated recursively and serves as the track quality measure.

There are two models for target existence propagation, Markov Chain One and Markov Chain Two. In the Markov Chain One model, the target at scan k can either exist (event χ_k) or not. If it exists, the target is always detectable with probability P_D. In Markov Chain Two model target may exist and be detectable (event χ_k^d), exist and be temporarily undetectable (event χ_k^n) of not exist. The probability of event χ is denoted with ψ.

Markov Chain One propagation formulae is used to update a-priori probability $\psi_{k|k-1}$ of target existence

$$\psi_{k|k-1} = \pi_{11}^1 \psi_{k-1|k-1} + \pi_{21}^1 (1 - \psi_{k-1|k-1}) \quad (13)$$

Markov Chain Two propagation formulae are given with

$$\begin{aligned} \psi_{k|k-1}^d = & \pi_{11}^2 \psi_{k-1|k-1}^d + \pi_{21}^2 \psi_{k-1|k-1}^n \\ & + \pi_{31}^2 (1 - \psi_{k-1|k-1}) \end{aligned} \quad (14)$$

$$\begin{aligned} \psi_{k|k-1}^n = & \pi_{12}^2 \psi_{k-1|k-1}^d + \pi_{22}^2 \psi_{k-1|k-1}^n \\ & + \pi_{32}^2 (1 - \psi_{k-1|k-1}) \end{aligned} \quad (15)$$

$$\psi_{k-1|k-1} = \psi_{k-1|k-1}^d + \psi_{k-1|k-1}^n \quad (16)$$

where $\pi_{i,j}^k; i,j = 1-3;\ k = 1,2$ denote the transitional Markov probabilities between the target existence states. For various reasons [**?**] we recommend $\pi_{21}^1 = \pi_{31}^2 = \pi_{32}^2 = 0$.

Although our study includes both Markov Chain One and Markov Chain Two, we will briefly outline only IPDA Markov Chain One equations. Both Markov Chain One and Markov Chain Two equations can be found in [**?**, **?**]. In the rest of this section IPDA will mean IPDA Markov Chain One. IPDA uses PDA formulae (1 - 5) with

$$\hat{m}_k = m_k - P_D P_G \psi_{k|k-1} \quad (17)$$

The a-posteriori probability of target existence is IPDA track quality measure and is calculated as

$$\psi_{k|k} = \frac{(1-\delta_k)\psi_{k|k-1}}{1 - \delta_k \psi_{k|k-1}} \quad (18)$$

3 False Track Discrimination Reliability

False track discrimination capability is a most important feature of the target tracking subsystem. This section will present a simulation study on reliability of the false track discrimination process of IPDA (both Markov Chain One and Markov Chain Two) and IMM-PDA.

False track discrimination can be performed by calculating the track quality measure and comparing it against thresholds. If the track quality measure rises above the confirmation threshold, the track is confirmed; i.e declared to be a true track. If the track quality measure falls below the termination threshold, the track is declared to be a false track and terminated. Confirmed tracks are presented to the operators and/or forwarded to the next level of processing. The thresholds can be constant or variable, for simplicity we use constant thresholds.

The simulation study presented in this paper addresses false track discrimination reliability with respect to the probability of detection, P_D, and sensor measurement covariance, R. Reliability with respect to other parameters, such as the clutter density and the target trajectory model mismatch, is not addressed.

For the purpose of comparison, it is assumed that, given a set of values for environmental, sensor and target parameters, each algorithm presented is optimized separately with respect to the termination threshold, the confirmation threshold and the initial value of track quality. The optimization criteria is to maximize the total number of confirmed true tracks, subject to the constraint of having predetermined and fixed total number of confirmed false track scans in the simulation experiment.

The purpose of the study is to determine

- performance penalty if the real value of P_D is smaller than the nominal P_D that the algorithm uses and is optimized for,
- performance penalty in the situation where we optimize for and use the worst case (lowest possible) value for P_D, instead of using the real value of P_D. In other words, the performance penalty when we decide to play it safe and always use the lowest possible value for P_D,
- performance penalty if real value of R is higher than the nominal R that the algorithm uses and is optimized for, and
- performance penalty in the situation where we optimize for and use the worst case (highest possible) value for R, instead of using the real value of R. In other words, the performance penalty when we decide to play it safe and always use the highest possible value for R.

3.1 Simulation Experiment

Each simulation experiment consists of 1000 runs, each run has 21 scans. A 2-dimensional surveillance situation was considered. The area under surveillance was $1000m$ long and $400m$ wide. The false measurements satisfied a Poisson distribution with density $1.0 \cdot 10^{-4}/scan/m^2$ over the area except for two patches with seven times this clutter density. The high clutter density patches are rectangular with corner coordinates $(x_{min}, x_{max}, y_{min}, y_{max})$ of $(330, 490, 203, 303)m$ and $(715, 840, 100, 200)m$. The target motion is modelled in Cartesian coordinates as

$$x(k+1) = Fx(k) + \nu(k) \tag{19}$$

where $x(k)$ is the target state vector at time k and consists of the position and the velocity in each of the 2 coordinates

$$x^T = [\, x \; \dot{x} \; y \; \dot{y} \,] \tag{20}$$

with the transition matrix

$$F = \begin{bmatrix} F_T & 0 \\ 0 & F_T \end{bmatrix}; \quad F_T = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \tag{21}$$

where T is the sampling period. The plant noise $\nu(k)$ is a zero mean white Gaussian noise with known variance

$$E\left[\nu(k)\nu(j)^T\right] = Q\,\delta(k,j) \tag{22}$$

where $\delta(k,j)$ is the Kronecker delta function and

$$Q = q \begin{bmatrix} Q_T & 0 \\ 0 & Q_T \end{bmatrix}; \quad Q_T = \begin{bmatrix} T^4/4 & T^3/2 \\ T^3/2 & T^2 \end{bmatrix} \tag{23}$$

with $q = 0.75$. The detection probability is constant throughout the experiment. Base sensor measurement noise covariance matrix is

$$R_b = \begin{bmatrix} 25 & 0 \\ 0 & 25 \end{bmatrix} \tag{24}$$

The scan interval (sampling period) is $1s$. The track estimation filter is a simple Kalman filter based on the described trajectory and sensor models. IMM-PDA and IPDA Markov Chain One use transitional probabilities

$$\begin{bmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{bmatrix} = \begin{bmatrix} \pi_{11}^1 & \pi_{12}^1 \\ \pi_{21}^1 & \pi_{22}^1 \end{bmatrix} = \begin{bmatrix} 0.98 & 0.02 \\ 0 & 1 \end{bmatrix} \tag{25}$$

and IPDA Markov Chain Two uses transitional probabilities

$$\begin{bmatrix} \pi_{11}^2 & \pi_{12}^2 & \pi_{13}^2 \\ \pi_{21}^2 & \pi_{22}^2 & \pi_{23}^2 \\ \pi_{31}^2 & \pi_{32}^2 & \pi_{33}^2 \end{bmatrix} = \begin{bmatrix} 0.90 & 0.08 & 0.02 \\ 0.28 & 0.70 & 0.02 \\ 0 & 0 & 1 \end{bmatrix} \tag{26}$$

Tracks were initialized automatically using two point differencing procedure outlined in [**?**], with the initial probabilities of target existence assigned according to [**?**]. The optimization constraint is that the sum of confirmed false track scans in each experiment be approximately 420.

3.2 Reliability with respect to the probability of detection

In this subsection we denote the nominal value of P_D (the value used in the track update calculations) with P_D^n; the actual value of P_D (the value used when generating target measurements during simulations) with P_D^a. In this draft version of the paper, only IPDA Markov Chain One results are presented; the final version will present the complete set of results.

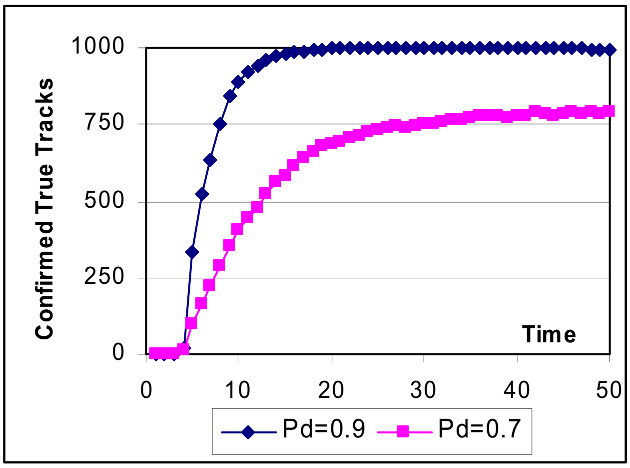

Fig. 1: Optimistic P_D

Figure 1 shows the effect of using optimistic value for probability of detection, P_D, during track update for IMM-PDA, and IPDA Markov Chain One and Two. Two curves are shown for each of the 3 algorithms:

- $P_D^a = P_D^n = 0.9$
- $P_D^a = 0.7;\; P_D^n = 0.9$

This will determine the extent of performance penalty due to mismatched value of P_D^n. One possible strategy is the "worst case" strategy in which, for the nominal value of P_D we use the lowest value for which we want to be able to track the target.

Figure 2 shows the penalty of using the "worst case" strategy with respect to P_D. Two curves are shown for each of the 3 algorithms:

- $P_D^a = P_D^n = 0.7$
- $P_D^a = 0.9;\; P_D^n = 0.7$

Comparing the second curve of Fig. 2 with the first curve of Fig. 1 we determine the performance penalty of the "worst case" strategy with respect to P_D.

3.3 Reliability with respect to sensor measurement noise covariance

In this subsection we denote the nominal value of sensor measurement noise covariance matrix R (the value used in the track update calculations) with R^n, and the actual value

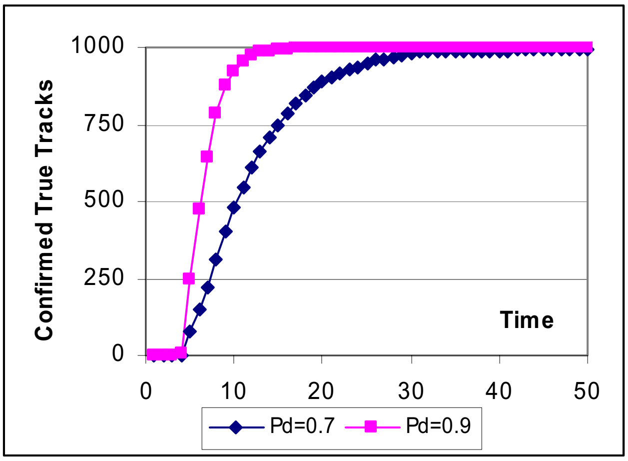

Fig. 2: Pesimistic P_D

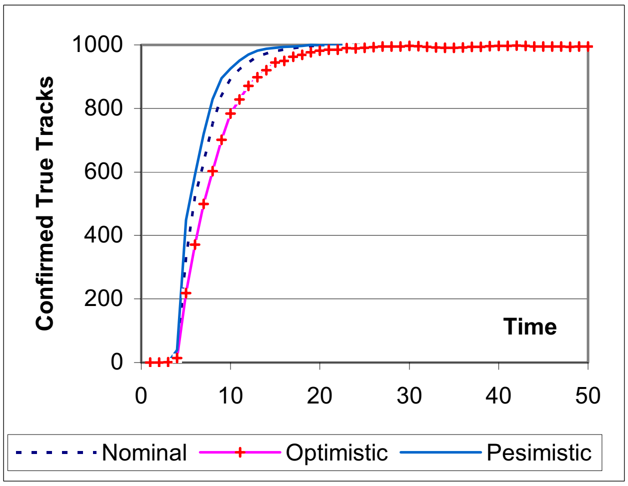

Fig. 3: Optimistic R

of R (the value used when generating target measurements during simulations) with R^a.

Figure 3 shows the effect of using optimistic value for sensor measurement noise covariance, R, during track update for IMM-PDA, and IPDA Markov Chain One and Two. Two curves are shown for each of the 3 algorithms:

- $R^a = R^n = R_b$
- $R^a = 1.5\ R_b;\ P_D^n = R_b$

This will determine the extent of performance penalty due to mismatched value of R. As with the probability of detection reliability, one possible strategy is the "worst case" strategy in which, for the nominal value of R we use the highest value we can reasonably expect.

Figure 4 shows the penalty of using the "worst case" strategy with respect to R. Two curves are shown for each of the 3 algorithms:

- $R^a = R^n = 1.5\ R_b$
- $R^a = R_b;\ R^n = 1.5\ R_b$

Image here

Fig. 4: Pessimistic P_D

Comparing the second curve of Fig. 4 with the first curve of Fig. 3 we determine the performance penalty of the "worst case" strategy with respect to R.

4 Conclusions

In this paper we have examined some reliability properties of IPDA and IMM-PDA algorithms for target tracking in clutter. Probability of detection and measurement noise covariance matrix are usually unknown and/or time varying. Based on a limited simulation study, we have attempted to determine the sensitivity of target tracking performance to using incorrect values, as well as the possible cost of the "worst case" strategy.